Journal of Applied Econometrics and Statistics
Vol. 1, No. 1, 2022, pp. 23-30
https://DOI: 10.47509 /JAES.2022.v01i01.03

ARF INDIA
Academic Open Access Publishing
www.arfjournals.com

A Comparative Analysis of Tests for the Presence of ARCH in Conditional Heteroscedastic Models

[1]Semiu Ayinla Alayande and [2]Ankeli Uchechi. C

[1]*Department of Mathematical Sciences, College of Natural Sciences, Redeemer's University, Ede, Osun State Nigeria. E-mail: alayandesa@run.edu.ng*
[2]*Federal Polytechnic Ede, Ede, Osun State, Nigeria. E-mail: Cosuche2010@gmail.com*

Received: 21 February 2022; Revised: 22 March 2022;
Accepted 15 April 2022; Publication: 30 June 2022

Abstract: The most successful GARCH model appears to be GARCH (1,1)model has an excess kurtosis greater than 0 and its distribution is therefore also heavily-tailed as in the ARCH(1) case. Asymptotic theory and estimation for those tests does not have the true log-likelihood for error term and therefore the estimates obtained are only quasi-likelihood estimates. The paper studied three tests using Monte Carlo experiment and obtained that LM test is the best to use unless the necessary criteria are not available and cannot be derived.

Kewwords: Heteroscedasticity, Monte Carlo, GARCH models, Conditional variance

Cl: M60

1. INTRODUCTION

The need to model data in economics and in particular, in finance where heteroscedasticity is the norm brings out the issue of autoregressive conditional heteroscedasticity model. The autoregressive moving average (ARMA) model with Gaussian noise and constant variance is inadequate in describing such data. The classical solution to the heteroscedasciticy problem is to assume that the variance is given by pool variance where the pool is an exogenous variable. This solution is unsatisfactory as argued by Engle (1982) in the time series context as it fails to recognize that the variance, like the mean, can also evolve over time.

The conditional variance is independent on square of error term which is a shock noise to the time series. Hence a large shock of lag error term will lead to a larger conditional variance for the error term.

for citation :

Semiu Ayinla Alayande and Ankeli Uchechi. C. (2022). A Comparative Analysis of Tests for the Presence of ARCH in Conditional Heteroscedastic Models. *Journal of Applied Econometrics and Statistics,* Vol. 1, No. 1, pp. 23-30. https://DOI: 10.47509 / JAES.2022.v01i01.03

The ARCH model were first applied to study the variance of UK inflation by Engle (1982). Bollerslev, chou, and Kroner (1992) and Booerslev, Enge, and Nelson (1994) are the two earlier reviews. Also, Li, Ling, and McAleer (2002) also work on these. Boollerslev extended the ARCH (q) process by including lagged values variance. The most successful GARCH model appears to be GARCH (1,1)model has an excess kurtosis greater than 0 and its distribution is therefore also heavily-tailed as in the ARCH(1) case. Asymptotic theory and estimation for ARMA-ARCH models given by Weiss (1986).Weiss also studied the case where the log-likelihood is not the true log-likelihood for error term and therefore the estimates obtained are only quasi-likelihood estimates.

The performance of tests as propounded by different author is the main focus of this paper. The test studied are; A Lagrange multiplier (LM) test with a portmanteau equivalent; Lee and Kings test and HJongs test. These tests checks for the presence of ARCH in conditional heteroscedasticity models.

1.1. A Lagrange Multiplier (LM) Test

Engle (1982) originally derived an LM test for the presence of ARCH. Let be the residual from a least square fits of the model.

$$y_t = \theta_0 + \theta_1 y_{t-1} + \ldots + \theta_p y_{t-p} + \varepsilon_t, \quad where \;\; t = 1 \ldots q \ldots . \propto \cdots \;\ldots \ldots \ldots \ldots \ldots \ldots$$

Let $z_t = \left(1, \hat{\varepsilon}_{t-1}^2 \ldots, \hat{\varepsilon}_{t-q}^2\right)$ and let ht, where α is a $(q+1)$ vector of parameters. Under the null of no autoregressive conditional heteroscedasticity h_t is a constant equal to h_0.

Assuming a normal ε_t, LM is asymptotically χ_q^2 distributed under the null hypothesis of no ARCH. An asymptotically equivalent form of LM can be obtained regressing $\hat{\varepsilon}_t^2$ on $(1, \hat{\varepsilon}_{t-1}^2 \ldots, \hat{\varepsilon}_{t-q}^2)^T)$ and then the test is given by $n.R^2$, the coefficient of determination of this regression. Lee(1991) showed that the LM test is infact equivalent to that of testing the same null hypothesis against an ARCH(q) process as the alternative.

1.2. Lee and King's Test

The LM test for the null of no ARCH against the alternative of an ARCH process ignores the inequality constraint for α_i and β_i. It is natural to ask whether a test with these constraints taken into consideration would have better performance in terms of size and power.

Let $y_t = \varepsilon_t = \sqrt{r_t a_t}$ and consider a statistical model involving the vector parameter $\theta = (\theta_1^T, \theta_2^T)$ and H_0: $\theta_2 = 0$ but the alternative R_A is now that at least one of the elements is greater than zero. A one-sided LM test could be based on the statistics $T - \hat{K}/(i^T(\hat{I}^{22})^{-1}i)^{1/2}$ where I^{22} denotes the lower h x h block of inverse of the Fisher information matrix, and $\hat{I}^{22}$ is the value of I^{22} evaluated at $\hat{\theta} = \left(\theta_1^T \, 0^T\right)^T$. Similarly $\hat{K}$ is the value of K evaluated at $\hat{\theta}$; is an r x 1 vector of ones. The K_{ARCH} is given by

$$K_{ARCH} = \frac{(n-p)\sum_{t=q+1}^{n}(y_t^2/h_0 - 1)\sum_{i=1}^{q} y_{t-i}^2}{\left\{2\sum_{t=q+1}^{N}\left(\sum_{i=1}^{q} y_{t-i}^2\right)^2 - 2\left(\sum_{t=q+1}^{n}\sum_{i=1}^{q} y_{t-i}^2\right)^2/(n-p)\right\}^{1/2}}$$

Under H_0, K_{ARCH} is asymptotically N(0,1) distributed so that one end-sided test can be easily applied.

1.3. A Rank Portmanteau Statistic

With the possible of presence of outliers rank autocorrelations are attractive non-parametric alternatives to standard autocorrelation coefficients.

The rank autocorrelation at lag k for a time series $\{y_1, \ldots, y_n\}$ is given by

$$\tilde{r}_k = \frac{\sum_{t=k+1}^{n}(G_t - \bar{G})(G_{t-k} - \bar{G})}{\sum_{t=1}^{n}(G_t - \bar{G})^2}, \qquad 1 \le k \le n-1$$

Where G_t is the rank of observation y_t, with

$$\bar{G} = \sum_{t=1}^{n} G_t / n = n(n+1)/2$$

$$\sum_{t=1}^{n}(G_t - \bar{G})^2 = n(n^2 - 1)/12$$

$$E\left(\tilde{r}_k\right) = -n(n-k)/n(n-1)$$

The variance of $\tilde{r}_k$ is given by

$$\operatorname{var}(\tilde{r}_k) = \frac{5n^4 - (5k+9)n^3 + 9(k-2)n^2 + 2k(5k+8)n + 16k^2}{5(n-1)^2 n^2 (n+1)}$$

Such that $1 \le k \le n-1$

Adapt from Dufour and Roy (1986). Let $\mu_k = E(\widetilde{r_k})$ and $\tilde{\sigma}_k^2 = \mathrm{var}(\widetilde{r_k})$ this led to statistics as shown by Dufour and Roy

$$D_R = \sum_{k=1}^{M} \frac{\left(\widetilde{r_k} - \mu_k\right)^2}{\tilde{\sigma}_k^2}, \qquad k = 1 \ldots\ldots\ldots\ldots\ldots\ldots M$$

Follows a χ_M^2 distribution asymptotically.

2.1. Derivation of the Model

Consider $y_t = \phi_0 + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \varepsilon_t$

Where ε_t is an ARCH(q) or GARCH(p,q) process t?

Consider the Lemma:

Let Y_t be a stationary and ergodic time series. Let F_t be the information set ($\sigma - field$) generate by all past observations up to and including time t. For simplicity, F_t is generated by $\{Y_t, Y_{t-1}, \ldots\ldots\}$ only. Given F_{t_1}, the distribution of Y_t is assumed to be Gaussian with conditional mean $\mu(\theta : F_{t-1})$ and conditional variance $h(\theta : F_{t-1})$, where θ is an l X 1 vector of parameters.

Let $\mu_t = \mu(\theta; F_{t-1})$ and $h_t = h(\theta; F_{t-1})$ for convenience. Both μ_t and h_t are assumed to be known except for the parameter θ and they are both assume to have continuous second-order derivatives almost surely.

Assume the model is of the form

$$Y_t = \phi Y_{t-1} + \varepsilon_t$$

With

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2$$

The best way to know the performance of each test is to consider their size and power. This was resolved through Monte Carlo experiment.

Generating the data X_t by a random walk without a drift:

$$X_t = X_{t-1} + \eta_t, \quad \eta_t \sim N(0,1)$$

Y_t is defined as

$$X_t = X_t + V_t,$$

Where V_t is an AR (1) process,

$$V_t = \rho V_{t-1} + w_t, \qquad w_t \sim N(0, \sigma_w^2)$$

3.1. SIMULATION RESULTS

The simulation results of the experiments and selected test for time series equation are described below. The choice of values for ñ is motivated by earlier studies conducted by Hipel and McLeod (1994). The analysis follows the criteria proposed by Alayande, S.A (2018) using Monte Carlo experiment. All results reported below are for a 10% significance level. The results reported in Tables 3.1 to3.3 are based on explanatory variable x for DGP with. w=1

The relative performance of each test and focus are discussed first, for the L&K test, then the LM test and the PS test follows. In the tables below, when ρ equals 0.99, and the equivalent σ is 1, the following results were obtained. L&K recorded highest value of 0.068 when η is 0 and lowest value of 0.024 when η is –0.8. LM has highest value of 0.388 when η is –0.1 and lowest value of 0.259 when η is –0.8. PS has highest value of 0.511 when η is –0.8 and lowest value of 0.175 when η is 0.2.

In summary on compare together, for $\sigma = 1$ in table 3.1, the power of all tests is relatively low and the null hypothesis of no ARCH is not often enough rejected. The PM test and L&K test have the lowest power and, on the hand, the LM test have the highest powers. When η deviates from zero, the power of all tests decreases (especially for negative values of η except for the (LM) test that shows an increase in power. Hence LM test is the best to use unless the necessary criteria are not available and cannot be derived.

3.2. Application to Nigeria Monthly Money Supply

A real-world data set is analyzed in this section to demonstrate the application of the proposed method. Specifically, with the observed data of the monthly money supply to the economy from January 1981 to December 2020.

Figure 1.1 shows time plot of the monthly money supply in billions M1) during the period from January 1981 to December 2020. Nigeria economy experience highest supply of money supply in 2006 from the data supply.

Figure 2 and 3 shows the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the first difference of M1 series. Standard Box-Jenkins show that the differenced series is stationary and can be fitted by an MA (1) model.

The three tests; LM test, Lee and King's test and ran portmanteau test are applied to the residual of both series. The degree of freedom considered are 1, 5, 9 and 15. The results are summarized in the table 1 below. The smoothed M1 series results indicate clearly that the data contain conditional heteroscedasticity, whereas results for the M1 series shows that the LM test detects nonlinearity unambiguously while the other two test fail.

Figure 1

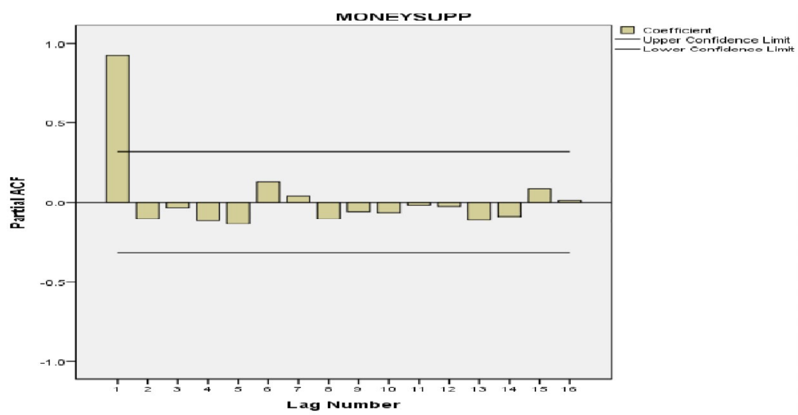

Figure 2

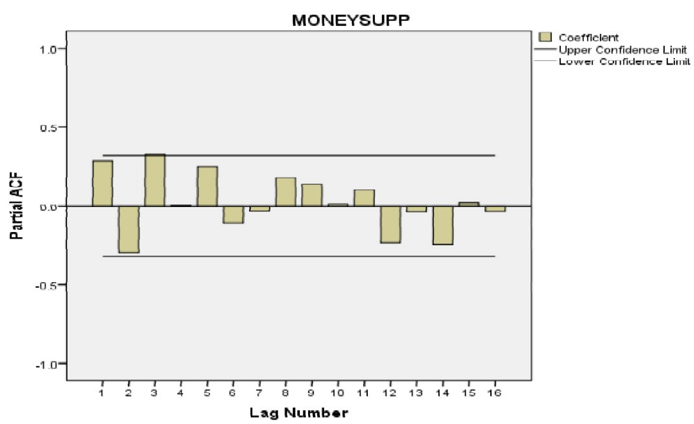

Figure 3

Table 4.1: Comparison of the three test as apply to Money Supply

Test/Variable	*DF*	*LM test*	*Lee and King's test*	*Rank Portmanteau statistics*
Money Supply (M1)	1	28.987	2.766	0.654
	5	36.766	4.589	0.874
	9	48.898	5.763	0.693
	15	67.543	5.787	6.435
Smoothed M1	1	12.212	2.808	1.569
	5	33.764	33.421	3.905
	9	50.871	45.687	8.265
	15	88.900	63.654	12.779

CONCLUSION

In conclusion, application to Nigeria economy (money supply) shows that LM test clearly portrayed what happened in economy through money supply. It shows that less money supply can keep inflation in check.

REFERENCES

Alayande S.A (2018). Why Segregating Cointegrating tests? *American Journal of Applied Mathematics and Statistics.* Vol. 6. No 4, 121-125.

Bollerslev, T., Chou, R.Y, and Kroner, K.F (1992). ARCH modeling in finance, a review of the theory and empirical evidence. *Journal of Econometric*, 52, Pg. 5-59.

Bollerslev, T., Engle, R.F, and Nelson, D.B (1994). ARCH models in Handbook of Econometrics 4Ed. R.F. Engle and D.L. McFadden, North-Holland, Amsterdam.

Dufour, J.M. and Roy, R. (1986). Generalised portmanteau statistics and tests of randomness. *Commun Statist-Theory*, *Method* 15, Pg 2953-2972.

Engle, R.F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation. Econometrica, 50, 987-1008.

Hipel, K.W. and McLeod, A.I. (1994). Time Series Modelling of Water Resources and Environmental Systems. Elsevier Science: Amsterdam.

Li, W.K., Ling, S. and McAleer, M. (2002). Recent theoretical results for time series models with GARCH errors. *Journal of Economic Surveys*, 16, Pg: 245-269.

Wong C.S. and Li. W.K (2001). On a mixture Autoregressive conditional Heteroscedastic Model. J. America Statistical Association. Pg 96, 982-995.

Wong H and Li. W.k (1996). Distribution of the cross -correlation of Squared residuals in ARIMA models. *The Canadian Journal of Statistics*, 24; 489-502.

APPENDIX

Table 3.1: Power of 10% level test with the null hypothesis of no ARCH when (ρ = 0.99)

σ =1											
η	-0.5	-0.4	-0.3	-0.2	0.1	0	0.1	0.2	0.3	0.4	0.5
L&K	0.338	0.345	0.359	0.368	0.374	0.388	0.337	0.332	0.315	0.302	0.301
LM	0.422	0.384	0.376	0.372	0.371	0.176	0.251	0.278	0.293	0.341	0.241
PS	0.053	0.054	0.054	0.057	0.058	0.061	0.032	0.031	0.030	0.029	0.054

Table 3.2: Power of 10% level test with the null hypothesis of no ARCH when (ρ = 0.99)

σ = 2											
η	-0.5	-0.4	-0.3	-0.2	0.1	0	0.1	0.2	0.3	0.4	0.5
L&K	0.339	0.343	0.362	0.369	0.374	0.385	0.366	0.354	0.325	0.318	0.309
LM	0.311	0.304	0.286	0.265	0.221	0.176	0.182	0.194	0.241	0.253	0.256
PS	0.063	0.065	0.066	0.068	0.071	0.073	0.071	0.069	0.054	0.052	0.042

Table 3.3: Power of 10% level test with the null hypothesis of no ARCH when (ρ= 0.99)

σ = 3											
η	-0.5	-0.4	-0.3	-0.2	0.1	0	0.1	0.2	0.3	0.4	0.5
L&K	0.267	0.277	0.285	0.301	0.308	0.311	0.282	0.281	0.254	0.248	0.243
LM	0.205	0.195	0.187	0.154	0.122	0.111	0.146	0.158	0.248	0.235	0.231
PS	0.042	0.043	0.045	0.046	0.047	0.048	0.045	0.044	0.244	0.269	0.285